# Universality in the Gross-Neveu model

Tomasz Korzec

Humboldt Universität zu Berlin

in collaboration with

Francesco Knechtli, Björn Leder, Ulli Wolff

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Outline

- Motivation
- The Gross-Neveu model
- Large $N$ expansion
- Conclusions
- Outlook: Monte Carlo

# Motivation

Results obtained with different actions should agree in the continuum limit

- Worrying results [S. Aoki @ Lattice 2000]
- Much work has been done to clarify, e.g.
  - ⋆ [D. Adams 2003] → today's plenary talk
  - ⋆ [BGR collaboration 2003]
  - ⋆ [B. Bunk, M. Della Morte, K. Jansen, F. Knechtli 2004]
    → this session
- Situation still not entirely clear [K. Jansen @ Lattice 2003]
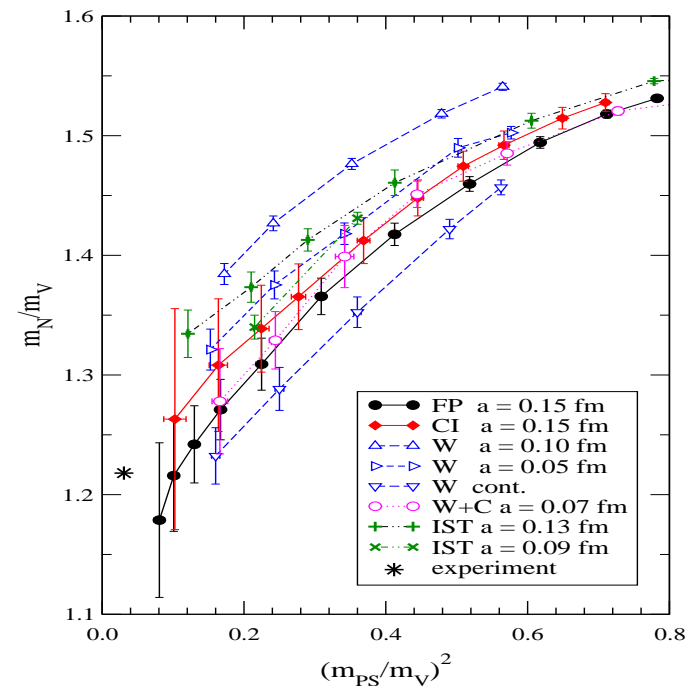


⇒ Our goal:

Compare continuum limits of different formulations by a high precision MC-study in a simple 2d model.
Here: results of large-$N_f$ calculations.

# The Gross-Neveu model

Lagrangian:

$$\mathcal{L}^{\mathrm{GN}} = \bar{\psi}^{(n)} \slashed{\partial} \psi^{(n)} - \frac{g_0^2}{2} (\bar{\psi}^{(n)} \psi^{(n)})^2, \qquad n = 1 \ldots N_f$$

- Renormalizable in two dimensions
- Asymptotically free
- Hidden $O(2N_f)$-symmetry $\Rightarrow$ stable under renormalization
- Spontaneously broken discrete chiral symmetry $\psi \to \gamma_5 \psi, \ \bar{\psi} \to -\bar{\psi}\gamma_5$
  $\Rightarrow$ dynamical mass generation
- Large-$N_f$ expandable

$\Rightarrow$ In many respects similar to QCD

Equivalent formulation:
$$\mathcal{L}^{\sigma\mathrm{GN}} = \bar{\psi}^{(n)} [\slashed{\partial} + \sigma] \psi^{(n)} + \frac{\sigma^2}{2g_0^2}$$

[D. Gross, A. Neveu '74; P. Mitter, P. Weisz '73; R. Dashen, B. Hasslacher, A. Neveu '75]

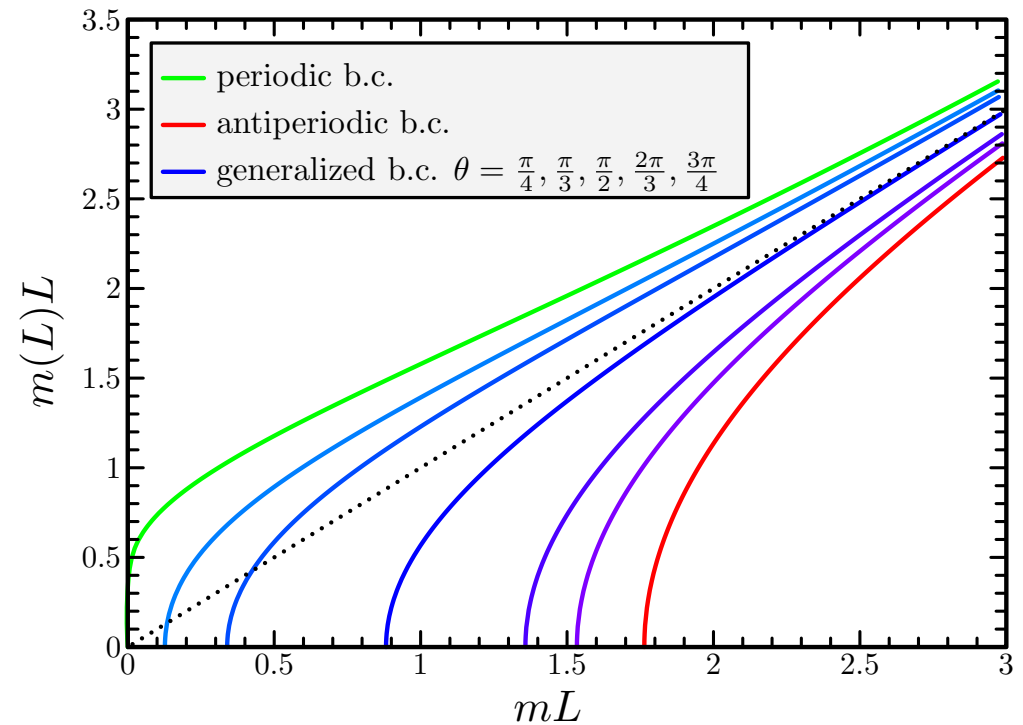# Gross-Neveu model in finite volume

From leading order $1/N_f$ expansion

- Spontaneous symmetry breaking of $\gamma_5$-symmetry
  (Discrete symmetry $\longrightarrow$ No conflict with Mermin-Wagner-theorem)
- Fermions aquire a mass $m$
- In finite volume mass is $m(L)$

We study the dimensionless quantity $m(L)L$ similar to the LWW-coupling in
$O(N)$ nonlinear $\sigma$ models
[Lüscher, Weisz, Wolff '86]

> The curve $m(L)L$ versus $mL$ is universal

# Large-$N_f$ expansion: the continuum theory

- $m(L) \to m$ for $L \to \infty$

- Qualitatively different behavior with periodic / antiperiodic b.c.

- With antiperiodic b.c.: Phase transition at
  $mL_c = \pi e^{-\gamma_E} \approx 1.764$
  [U. Wolff '85]

- With generalized b.c. (phase $e^{i\theta}$): $mL_c$ continously reaches $0$ when $\theta \to 0$



Legend:
- periodic b.c.
- antiperiodic b.c.
- generalized b.c. $\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}$

Axes: $m(L)L$ versus $mL$
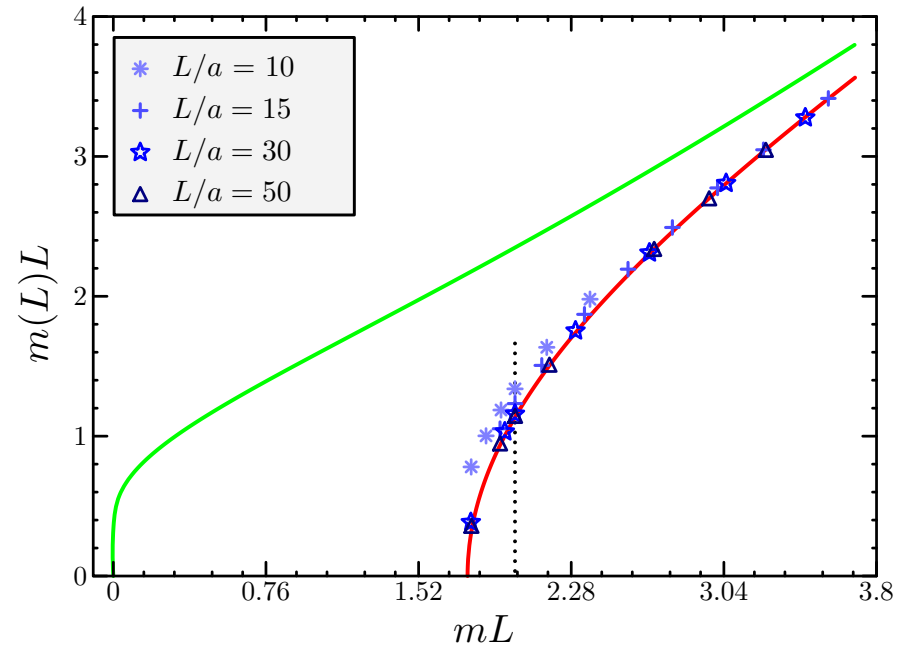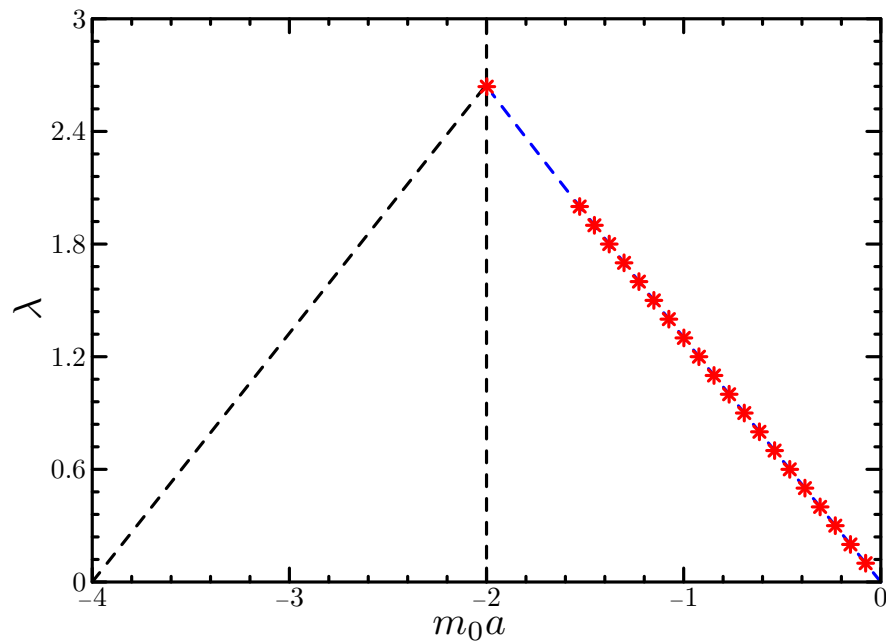
# Large-$N_f$ expansion: Wilson fermions

- Wilson term breaks $\gamma_5$-symmetry explicitly

- To recover it in the continuum: introduce bare mass parameter $m_0$ [S. Aoki, K. Higashijima '86]

- In leading order $1/N_f$ the effective potential is

$$V_{\text{eff}} = \frac{\sigma^2}{2\lambda} - \frac{1}{TL}\text{tr } \log(D_w + \sigma + m_0)$$

- It becomes symmetric with respect to the location of the maximum only in the continuum limit
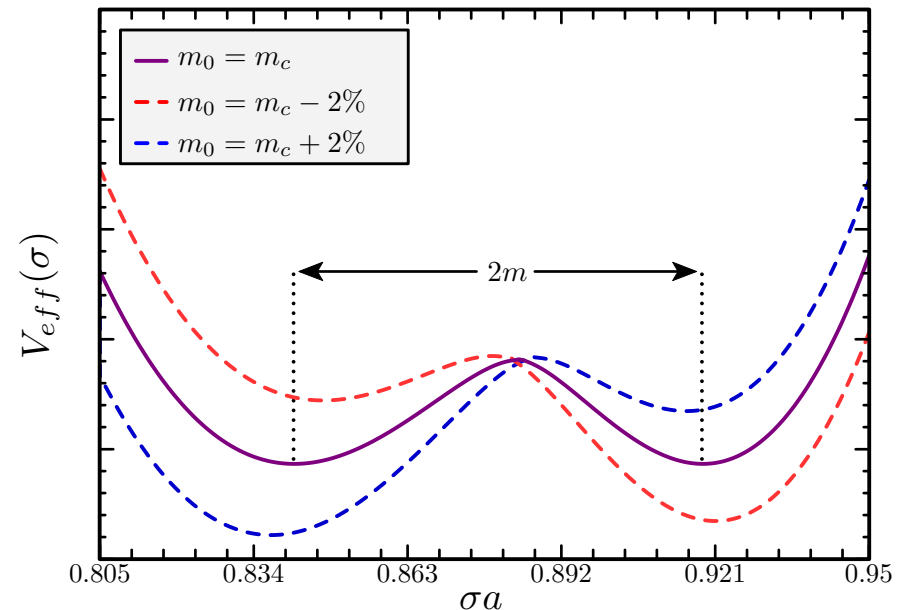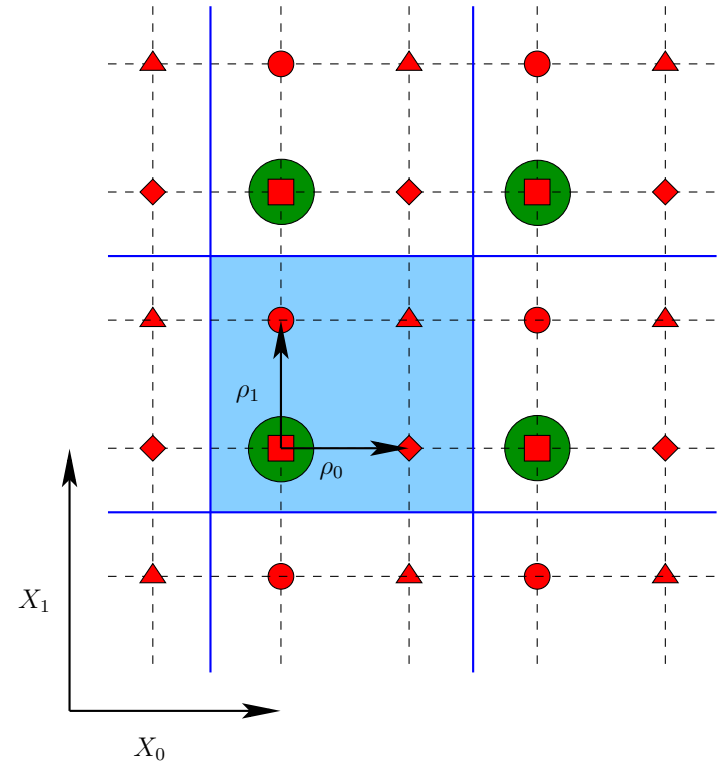
# Large-$N_f$ expansion: Wilson fermions



- Free Wilson fermions in $2d$: one massless mode, one with $m = 4/a$, two with $m = 2/a$
- Continuum results are well reproduced

# Staggered fermions

Action:

$$S = \sum_{x,\mu} \eta_\mu(x)\bar\chi(x)\tilde\partial_\mu\chi(x) + \sum_X \frac{\sigma(X)^2}{2g_0^2}$$

$$+ \sum_{X,\rho} 2\sigma(X)\bar\chi(2X+\rho)\chi(2X+\rho)$$



- Naive continuum limit: Gross-Neveu model with $2N_f$ flavors

- But: at finite $a$ no full $O(4N_f)$ flavor symmetry!

- An exact $\gamma_5$-symmetry for all lattice spacings
  $\Rightarrow$ No additive mass renormalization needed

[Y. Cohen, S. Elitzur, E. Rabinovici '82; T. Joliceur, A. Morel '86]
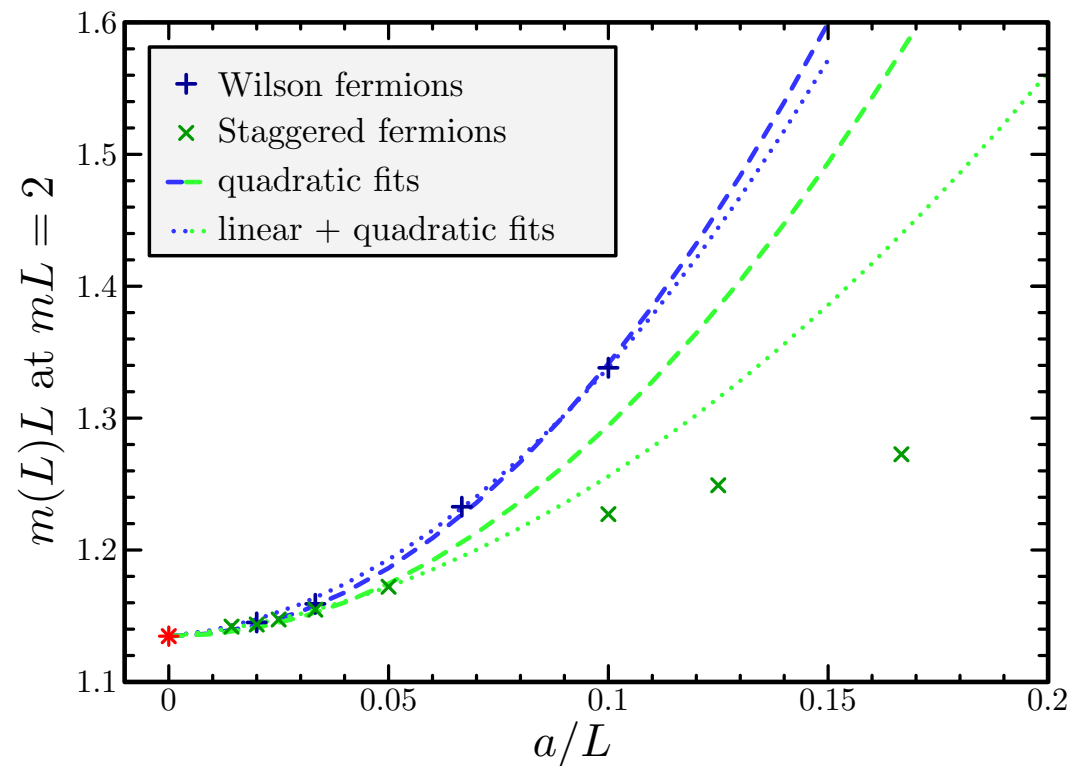
# Large-$N_f$ expansion: Staggered fermions

- To obtain the results in leading order of $1/N_f$ expansion it is necessary to solve a gap-equation

- With our staggered action this gap-equation is the same as that of a Gross-Neveu model discretized with naive fermions

$\Rightarrow$ Next to leading order computation could be worthwhile
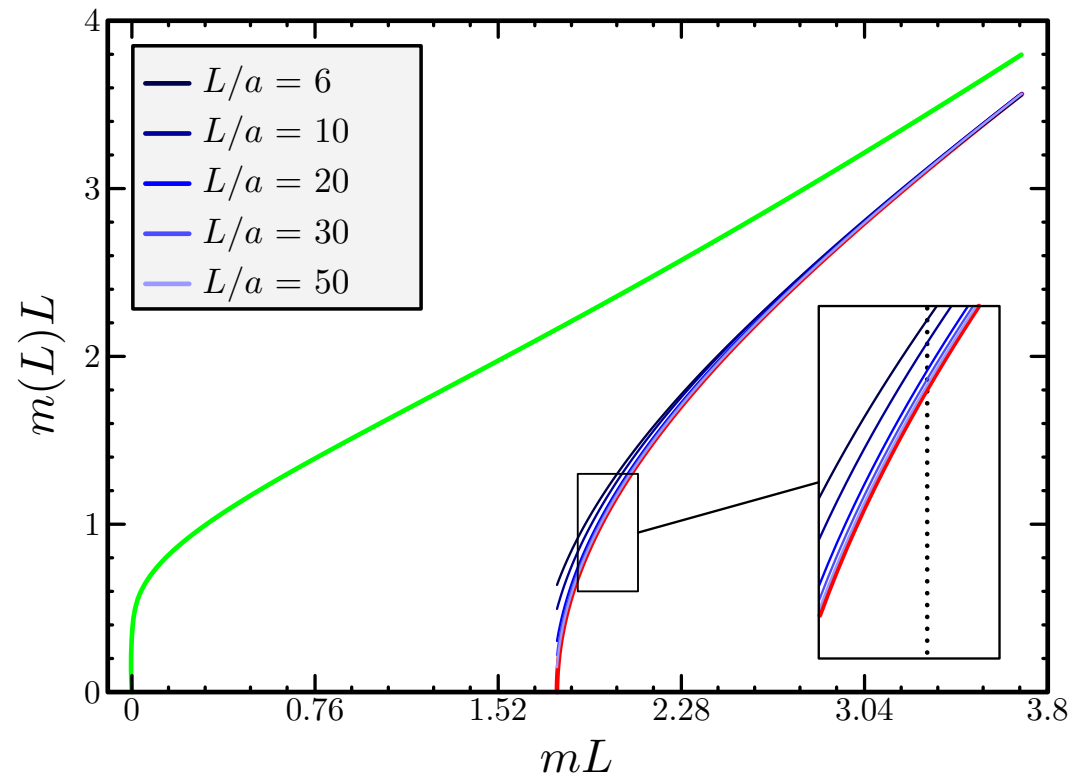
# Lattice artifacts

- In leading order $1/N_f$: both formulations are compatible with each other and with the continuum result.

- Approach to continuum
  - ⋆ Data suggests $O(a^2)$ for both formulations
  - ⋆ For a deeper understanding Symanzik's improvement programme has to be carried out (work in progress)

# Conclusions & Outlook

## Conclusions

- In leading order of the large-$N_f$ expansion Wilson- and staggered-fermions lead to compatible results
- But: method is quite insensitive to the number of doublers - in leading order $1/N_f$ even naive fermions work fine
- Already in this simplified setup some typical features of both formulations appeared
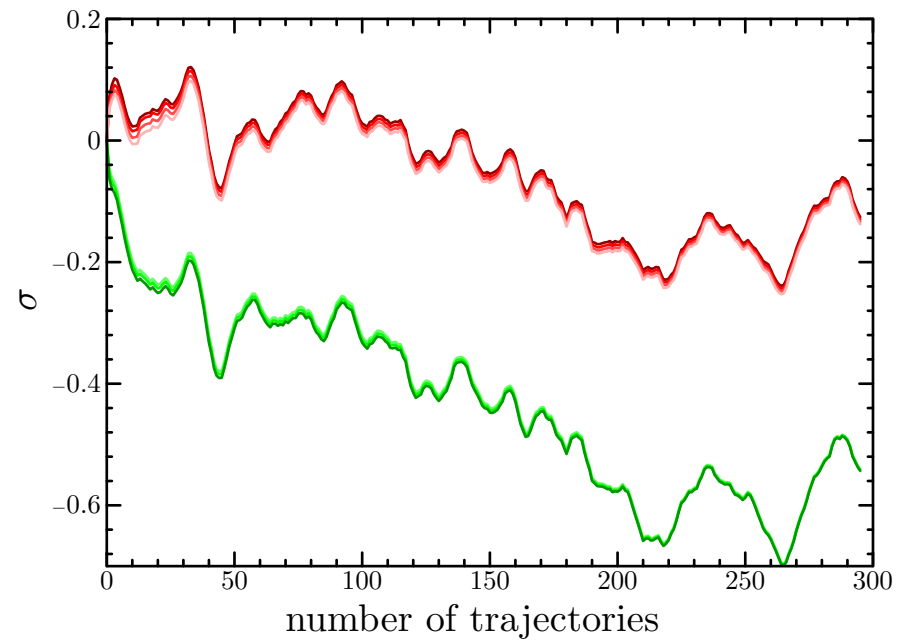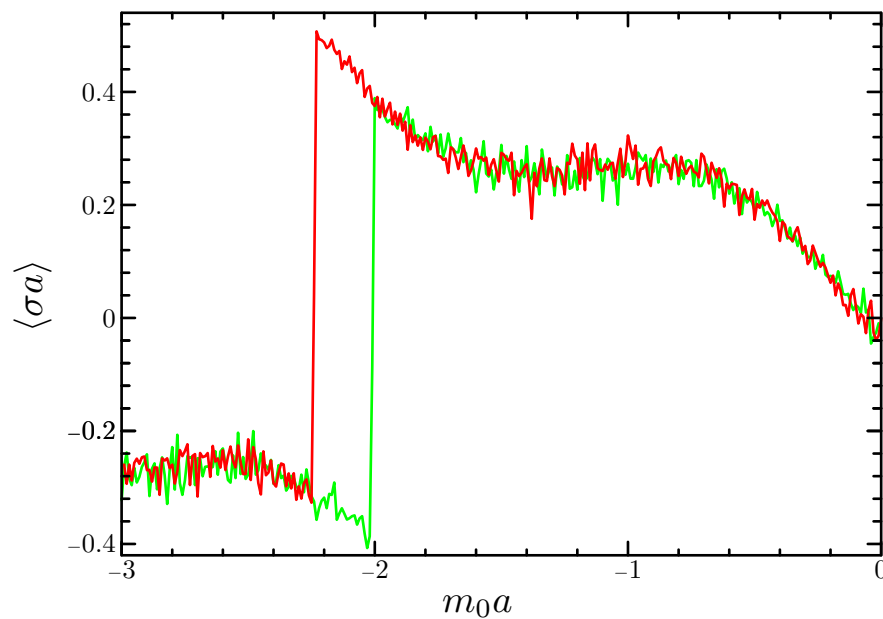
## Outlook

- High precision MC-study at finite $N_f$
- Symanzik Improvement
- Investigation of other fermion formulations

# Outlook: Monte-Carlo with Wilson fermions

Possible problems:

- No symmetry breaking in finite volume $\Rightarrow$ measure only invariant quantities
- How to determine $m_0$?



[M. Campostrini, G. Curci, P. Rossi '87; M. Beccaria, G. Curci, L. Galli '89]